The metric analysis of Diocletian Palace in Split has disclosed the practice of Roman modular proportioning in architectural composition. This still existing example of proportions and modules in Roman architecture can help to clarify the Vitruvius’ passages on proportio, commodulatio, and modulus. For this purpose the plan of the Palace, one of its elevations, and the eastern view of the Peristyle will suffice.

Measurement of plans and elevations of the Palace and of its rooms discloses the rhythm of various modules, identical with standard Roman sizes. (See illustration 1.) Thus, the plan of the Palace itself is in the rhythm of a module 5 passus large. (Illustration 3.) Similarly, the articulation of the Palace’s eastern elevation offers forms in sizes expressible with various Roman units of measurement, used as modules. The walls of intertura e.g. are in the rhythm of 12,5 passus, 12 passus, and 11 passus. (Illustration 4.) Dimensions of intertura themselves, of turrets, of windows and doors, would offer a number of modules ranging from 12,5 passus to 1 pes. (Illustrations 4, 5, and 6).

The seemingly equal rhythm of columns in Peristyle is from the point of view of modules (and proportion, as we shall see later) most interesting. The intercolumnium opposite to entrance of the Mausoleum is without the parapet and the opening is consequently higher than openings of other intercolumnia, diminished by the parapet. (Illustration 7.) The modules of the larger and of the smaller intercolumnia are 10 and 9 palmi respectively.

Evidently, sizes of the Diocletian Palace and of its members are multiples of various modules. Small members (windows e.g.) are measured by small modules; sizes of large members (interturria e.g.) are composed of large modules. It could be said that modules are proportionate to the composition and that modular multiples are generally small integers, which corresponds with the basic law of modular composition.

Modular multiples in dimensions of Diocletian’s Palace are identical with terms of Pell series and their ratios are consequently approximations to proportions of octagram. (Illustration 2.) Indeed, the thorough analysis of the Palace and its rooms discloses only the forms derived from the octagram, i.e. the square or the ratio 1 : 1,
the rectangles in the ratios $1 : \sqrt{2}$, $1 : 2$, and $1 : \frac{1+\sqrt{2}}{2}$, the cross and the octagon itself. No other shapes are found in Palace's plan and elevations. Even the mosaic patterns follow the rule of octagram. Thus, the Palace's plan e.g. is in the proportion called quadriagon, formed by the ratio $29 : 24$, nearing the $(1+\sqrt{2}) : 2$. (In this paper we do not enter into discussion about the proportional correction lengthening eastern and western elevation for half a module and enlarging southern side for one module.) On schemes of the eastern elevation No. 4, 5, and 6, ratios $1 : \sqrt{2}$, $1 : 2$, and $1 : \frac{1+\sqrt{2}}{2}$ are illustrated. Repetition of a ratio, or in other words, a constant ratio, is proportion. In our case we do not have only one ratio, but related ratios, all generated arithmetically in Pell series and geometrically in octagram. In this case we speak about a family of proportions.

To summarize: Dimensions of Diokletian Palace and of its parts are formed by whole multiples of various modules, equalling standard Roman sizes. Their ratios form constant related proportions. The understanding of principles of modular proportions in Diocletian Palace is a good help for the analysis of terms modulus, proportio, and commodulatio as used by Vitruvius. But it would exceed the scope of this paper to analyse such elements of Vitruvian design as ordinatio, dispositio, quantitas, eurythmia, symmetria, compositio, etc., though used in connection with modulus, proportio, and commodulatio. To clarify them thoroughly with the findings in the Diocletian Palace is possible; but for this purpose Palace's plan and one or two elevations only are not sufficient. So the words will be left unexplained or we will try to translate them in modern architectural jargon without long explanations.

The term modulus is first used by Vitruvius in the passage:

*Ordinatio est modica membrorum operis commoditas separation universeque proportionis ad symmetriam comparatio. Haec componitur ex quantitate, quae Graece posotes dicitur. Quantitas autem est modulorum ex ispius operis sumptio e singulisque membrorum partibus universi operis conveniens effectus.* (Book I. C. II 2).

Translation:

Order is a suitable disposition of members of a building and the coordination of standard sizes implying proportion for the single element and for the whole. It consists of dimensions, which is posotes in Greek. Dimension itself consists of modules from the building proper and from individual parts of members which results conveniently to the whole building.

According to Vitruvius proportion is linked to the separate part and to the whole. The eastern elevation of the Peristyle is a good example for the same proportion of the whole and of its parts: The colonade itself (universe) is in the ratio $7 : 3$. Its intercolumnia (separatim) are in the ratio $12 : 5$. Both ratios are analogous; they approximate the same proportion, called with the Greek letter $\Theta$:
Proportion of Diocletian Palace

7 : 3 \sim 12 : 5 \sim \Theta = (1 + \sqrt{2})

Comparatio ad symmetriam is equivalent to our coordination of sizes. Architect's job is to adjust dimensions. In the schemes of Diocletian Palace standard sizes (modules) are coordinated into related proportions.

The key word in our paragraph is modulorum sumptio (note the plural of modulus). If modulus means to us a common denominator of sizes, or a unit of measurement, it is obvious that a dimension equals several modules, unless it is the smallest one, equaling one unit only. The plural of modulus implies the number of modules (greater than one), but it is not clear if this plural means also a variety of modules, though this is hinted by the specification that modules come from two sides: from the building proper and from the parts of members. Knowing that a number of various modules forms the dimensions of Diocletian Palace, from the large town-planning module (5 passus) to the smallest module for a window (1 pes), we are ready to see in Vitruvius' plural not only the quantity greater than one but also the difference in sizes.

More specific about modulus, though he is not naming it, is Vitruvius in the following passage:

Item symmetria est ex ipsius operis membris conveniens consensus ex partibusque separatis ad universae figureae speciem ratae partis responsus. Uti in hominis corpore e cubito, pede, palmo, digito ceterisque particulis symmetros est eurhythmieae qualitas, sic est in operum perfectionibus. Et primum in aedibus sacrils aut e columnarum crassitudinis aut triglypho aut etiam embatere, ballista e foramine, quod Graeci peritreton vocitant, navibus interscalmio, quae dipechyaia dicitur, item ceterorum operum e membris inventur symmetriarum ratiocinatio. (Book I. C. II. 4).

This passage means to me:

System of measures is the adjustment of members of the building proper and the corresponding appearance of the whole figure arising from the selection of the unit of measures for every single part. As in the human body system of measures appears in the eurhythmy between cubit, foot, palm, finger, and other particles, so it is in the perfect buildings. The selection of standard units of measure can be found primarily in temples either from the thickness of a column, or from the triglyph, or from any other rhythmically repeated unit; in the balista, from the bore or peritreton, as called by the Greeks; in the ship, from the space between two tholepins, two cubits apart; and in other works, from their component members.

Ex partibus separatis ratae partis responsus means to me in modern language: correspondence or coordination of modules selected from single parts. The singular ratae partis does not mean one module only, since this is a generic genitive; here rata pars is relating to or characteristic of a whole group of separate parts. We know already
from Diocletian Palace that small parts are measured with small modules, large parts with large modules. And indeed in his next sentence Vitruvius is assuring us that human sizes acting as standard units of measurement appear as standard modules in perfect buildings. In other words, any Roman standard size can be used as a module. Accordingly, there is not such a thing as one only basic module.

Vitruvius lists examples what a *rata pars* derived from symmetria can be: either diameter of a column, or a (length of) triglyph, or *embatere*, all *partes* acting simultaneously as modules in the same temple. (*Embatere* means any rhythm in movement, such as pacing and spanning, or a size rhythmically repeated in such a movement.)

*Ballista* offers example of various modules for works of the same kind. Since *ballista* of a larger caliber must be stronger (larger), it is obvious that its module is larger. The module of *ballista* is linked to the diameter of the bore, or caliber.

The only example, where the module appears as an absolute size, is *navis*. The module for a boat follows the rhythm of rovers, spaced a *dipechyaia*, or two cubits, apart. The rows in modern cinema are usually 3' or 0,9 m distant. Human sizes are basically still not changed.

Terms *proportio* and *commodulatio* are mentioned together by Vitruvius in the following paragraph:

*Aedium compositio constat ex symmetria, cuius rationem diligentissime architecti tenere debent. Ea autem paritur a proportione, quae Graece analogiadicitur. Proportio est ratae partis membrorum in omni opere totiusque commodulatio, ex qua ratio efficitur symmetriarum. Namque non potest aedis ulla sine symmetria atque proportione rationem habere compositionis, nisi uti ad hominis bene figurati membrorum habuerit exactam rationem.* (Book III. C. I. 1)

Translation:

Composition of buildings is essentially composition of standard sizes, the ratios of which must be strictly observed by architects. It is generated by the proportion called *analogia* by Greeks. Proportion is selection of a unit of sizes for every member and for the whole of the building and their modular coordination, from which appears the ratio of standard sizes. For without standard sizes and proportion there is no rational composition for any building; it must follow exactly the ratio of members of a well shaped man.

If we look again at our illustrations we see that a scheme for Diocletian Palace is an abstract interplay of Roman standard sizes. Dimensions in architecture are decisive for building's function, construction, economy, and aesthetics. A room can not function if it is too large, or too small, or too wide, or too narrow, or too close, or too distant, etc.; its construction will not be statically safe if too thin, but it will be too expensive if too strong; that the size of a nose can influence the beauty of a face we know. Composition in architecture consists of sizes, or better, *symmetria*, which means standard sizes.
Any deviation from symmetria, or Roman standard sizes, is harmful: Vitruvius’ superlative *diligentissime* leaves no doubt. Since modern architecture does not know any equivalent to Roman standard modules, it must be emphasized that for a Roman architect *symmetria* was a powerful compositional tool. All distances in Diocletian Palace e.g. are rational multiples of one of the Roman standard units of sizes. Even in case when two architectural members are nearly equal, their sizes obey the Vitruvius’ precept. Two intercolumnia in Peristyle e.g. are but slightly different: the module of the larger intercolumnium is 1 gradus or 10 palmi; the module of the smaller intercolumnium is 9 palmi. But the proportion of axial intercolumniation in the Peristyle is constant, the height to the breadth being 12 : 5. Equal shape, or better, analogous shape, means proportion. Vitruvius’ statement that proportion is selection of a unit of measurement as a module for every member and for the whole is clear: in our case 10 and 9 palmi for adjacent intercolumnia. It is equally clear that modules for every member and for the whole of the building must be coordinated to get proportion: the colonade of the Peristyle is in the ratio 7 : 3 (module being 5 gradus) which is analogous to the ratio 12 : 5 of intercolumniation (with modules of 10 and 9 palmi).

Principles in proportioning, as found in Peristyle, seem to be condensed in Vitruvius’ words *Proportio est ratae partis* (a generic genitive again) *membrorum totiusque commodulatio*. This sentence sounds like a standard phrase in architectural jargon of Vitruvian times. It is perfectly true, but neither the technique nor the essence of proportion are disclosed by it. Now, the question is, how much did Vitruvius know about the know-how of the powerful guild of masons. If Vitruvius is one of the illuminated, his writing is obviously prepared for the general public (as we would say today; Vitruvius himself dedicated his books to Augustus). His books do not appear as manuals for professionals. If Vitruvius is one of the outsiders, his writing is only the common knowledge doing no harm to professional secret. Proportion and modular coordination are not the only point where such question arises.

Modular coordination and standard sizes, *cuius rationem architecti diligentissime tenere debent*, disappeared in Europe together with Roman standard sizes. In Gothic architecture they were substituted with geometrical proportioning (quadratura, triangulatura). Renaissance means the rebirth of module. Since in this period Roman symmetria was already lost, but replaced by the multitude of local sizes, module had to be fixed to something more constant than contemporary anthropometries, varying from town to town. So the column’s diameter became identified with the module. This is the origin of the teaching that module has only the aesthetic role and of the belief that in modern modular coordination one module is sufficient.
Back to Vitruvius. According to him there is indeed no rational composition without standard sizes and rational proportion, as found in a well shaped man. The names of Roman standard sizes have only a mnemonic function. Things become memorable when put in a meaningful order. The force of Roman standard sizes is hidden in their ratio. Units of *symmetria* are always in the ratios expressible with whole numbers, usually small, which follow exactly the basic law of modular composition.

Modern modular coordination is trying to find again, in words by Mark Hartland Thomas, the secretary of the British Modular society, the simplicity and directness of Roman building. For this goal the understanding of Roman *modulus*, *proportio*, and *commodulatio*, can be a great help.

*Ljubljana.*

T. Kurent.

**ILLUSTRATIONS**

1. Graphical and arithmetical presentation of Roman standard sizes, used as architectural modules in the largest span of space and time.

2. The plan of Diocletian Palace in modern Split on the town-planning grid in the rhythm of module M(5 passus). Modular multiples, 29 and 24, are terms of the first and of the double first Pell series. (English mathematician John Pell, 1610—1685, was the first in our civilization to describe the series, known before him by Romans, and named today after him). The ratio 29:24 approximates quadriagon or \((\sqrt{2}+1):2\). Quadriagon is one of the related proportions generated geometrically by the octagram.

3. Octagram, or eight pointed star, is the geometrical origin of related proportions:
   - 1 : 1 or square (prima),
   - 1 : 1,207 or quadriagon,
   - 1 : 1,414 or diagon,
   - 1 : 1,818 or dual diagon,
   - 1 : 2 or double square (octava),
   - 1 : 2,414 or double quadriagon (Θ).
   The listed proportions are approximated arithmetically by the ratios between terms of Pell series.

4. Proportion \(\sqrt{2}\) on the eastern elevation of Diocletian Palace. Ratios approximating \(\sqrt{2}\) on this elevation:
   - 3M(8 passus): 2M(8 passus), which is the size of the second interturrium from the left. First interturrium from the left is the largest, i.e. the highest and the longest: its module is for 5 trientes larger than 8 passus, which is due to proportional correction. The two interturria on the right are smaller, but still in the ratio 3:2;
   - 10M(1 pes): 7M(1 pes) is the typical window below the arch in the lateral interturria;
   - 7M(8 pedes): 5M(8 pedes) is the height to the width of the wall in the central interturrium;
   - 7M(gradus): 5M(gradus) is the size of the opening of Porta Argentea;
   - 3M(gradus): 2M(gradus) is the window below the arch in the central interturrium.
Modules are 8 passus, 8 pedes, 1 gradus, and 1 pes, large. Modular multiples, 2, 3, 5, 7, and 10, are terms of the first and second first Pell series. Their ratios 3:2, 10:7, and 7:5, approximate the proportion $\sqrt{2}$.

5. Proportion 2 on the eastern elevation of Diocletian Palace is approximated by the following ratios:
   - $2M(12.5 \text{ passus}) : 1M(12.5 \text{ passus})$,
   - $2M(12 \text{ passus}) : 1M(12 \text{ passus})$,
   - $2M(11 \text{ passus}) : 1M(11 \text{ passus})$, all sizes for walls in interturria from left to right;
   - $2M(8 \text{ passus}) : 1M(8 \text{ passus})$ for the turrets flanking Porta Argentea;
   - $2M(1 \text{ passus}) : 1M(1 \text{ passus})$, the size of the rectangle circumscribed to the arched window in the central interturrium;

2M(7 pedes) : 1M(7 pedes), the size of the rectangle circumscribed to the arched window in the lateral interturria;

Modules are 12.5 passus, 12 passus, 11 passus, 8 passus, 7 pedes, and 1 passus, large. Modular multiples, 1 and 2, belong to the first Pell series.

6. Proportion $\Theta$ on the eastern elevation of Diocletian Palace is approximated by the following ratios:
   - $5M(6 \text{ passus}) : 2M(6 \text{ passus})$ is the size of the wall in second interturrium from the left plus the adjacent part of the small tower. In dependance of the point of observation the small turret optically merges with the one or the other adjacent wall;
   - $5M(4 \text{ passus}) : 2M(4 \text{ passus})$ is the size of the south-eastern corner turret (first from the left);
   - $5M(3 \text{ passus}) : 2M(3 \text{ passus})$ is the size of the small turret on the right side.

Multiples 5 and 2 are terms of the first Pell series, their ratio approximates $\Theta$. Modules are 6 passus, 4 passus, and 3 passus.

7. Proportion $\Theta$ on the eastern side of Peristyle in Diocletian’s Palace is approximated by the following ratios:
   - $7M(5 \text{ gradus}) : 3M(5 \text{ gradus})$ is the length to height of the colonnade;
   - $10M(5 \text{ gradus}) : 4M(5 \text{ gradus})$ is the total length and maximal height of the Peristyle;
   - $12M(1 \text{ gradus}) : 5M(1 \text{ gradus})$ is the height to interaxial width of the intercolumnium leading from the Peristyle to the entrance to Mausoleum;
   - $12M(9 \text{ palmi}) : 5M(9 \text{ palmi})$ is the height to interaxial width of other intercolumnia, diminished by parapets.

Modules are 5 gradus, 1 gradus (or 10 palmi), and 9 palmi. Ratios of modular multiples 12:5, 10:4, (both from the first Pell series), and 7:5 (second Pell series), approximate $\Theta = 1 + \sqrt{2}$.

BIBLIOGRAPHY


| Figure 1 |

### Table 1: Distances in Roman Units

<table>
<thead>
<tr>
<th>Unit</th>
<th>Value</th>
<th>Value</th>
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<tbody>
<tr>
<td><strong>Decempeda</strong></td>
<td>295.74</td>
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</tr>
<tr>
<td><strong>Passus</strong></td>
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<tr>
<td><strong>Gradus</strong></td>
<td>73.90</td>
<td>cm</td>
</tr>
<tr>
<td><strong>Cubitus</strong></td>
<td>44.39</td>
<td>cm</td>
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<tr>
<td><strong>Palmpes</strong></td>
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<td><strong>Pes</strong></td>
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<td><strong>Quinques</strong></td>
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<td><strong>Triens</strong></td>
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<tr>
<td><strong>Sicilicus</strong></td>
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**Note:** Distances are given in centimeters (cm).
Proportion of Diocletian Palace

Fig. 2

PELLI SERIES

\[
\begin{align*}
1 &= 0 - \phi^0 - \phi^1 - \phi^2 - \phi^3 - \ldots \\
1 &= 2 - 5 - 12 - 29 - 70 - \ldots \\
2 &= 4 - 10 - 24 - 56 - 140 - \ldots \\
29 &= \times 0 (\text{QUADRIAGON}) = 0 \\
24 &= \times 0 (\text{MODULI}) = 0 \\
\theta &= \frac{f^2 + 1}{2} \approx 1,207
\end{align*}
\]

TURRIUM MURORUMQUE FUNDAMENTA SIC SUNT FACIENDA, UTI FODIANTUR, SI QUEAT INVENIRI, AD SOLIDUM ET IN SOLIDO, QUANTUM EX AMPLITUDINE OPERIS PRO RATIONE VIDEATUR, CRASSITUDINE AMPLIORE QUAM PARITETUM, QUI SUPRA TERRAM SUNT FUTURI, ET EA IMPLANTUR QUAM SOLIDISSIMA STRUCTURA.

VITRUVII DE ARCHITECTURA LIB. I. C. V

15 Živa Antika
\[ \Theta = 1 + 41 = 2,414... \]
\[ 41\Theta = 1 + 41 + 1 \]
\[ \Theta^2 = \Theta + 1 + 41 + 41\Theta^2 = 0 + 1 + 41 + 1 + \Theta \]
\[ \Theta^2 = 1 + 2\Theta = 3 + 2 \]
\[ \Theta^3 = 2 + 5\Theta = 7 + 5\Theta \]
\[ \Theta^4 = 5 + 12\Theta = 17 + 12\Theta \]
\[ \Theta^5 = 12 + 29\Theta = 41 + 29\Theta \]
\[ \Theta^6 = 29 + 70\Theta = 99 + 70\Theta \]

<table>
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<td>1 : 2</td>
<td>CQ</td>
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<tr>
<td>1 : 2,414</td>
<td>Θ</td>
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</table>
Fig. 4

PROPORTIO
1 : √2

MODULI
M 8 PASSUS
M 1 GRADUS
M 1'

NUMERI
1 - 2 - 5 - 12...
1 - 3 - 7 - 17...

Fig. 4
Fig. 5
Fig. 7

PROPORZIO

\( \theta = 1 + \sqrt{2} \)

NUMERI

\[
\begin{align*}
1: & = 2 - 5 - 12 - \\
2: & = 4 - 10 - 24 - \\
1 - 3: & = 7 - 17 - \\
10 \div 4: & = 7 - 15 - \theta = 1 + \sqrt{2} \\
\end{align*}
\]

MODULI

\[
\begin{align*}
V' & = 0 \\
V & = X \\
V & = XX \text{ GRADUS} \\
IV & = M_v \text{ GRADUS} \\
\end{align*}
\]